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LETTER TO THE EDITORS

SOME NOTES ON FORMULATION OF THE PROBLEM OF GAS FLOW IN A CIRCULAR TUBE

STEADY-STATE heat transfer of gas long tube flow at small Mach numbers ($M \ll 1$) and without considering the gravity is described by means of the boundary-layer type equations (1)–(3)

$$\frac{\partial \rho u}{\partial x} + \frac{1}{r}\frac{\partial \rho v r}{\partial r} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu \frac{\partial u}{\partial r}\right) \qquad (2)$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{k}{C_p}\frac{\partial H}{\partial r}\right) \qquad (3)$$

while initial and boundary conditions must be written in the following form

$$H(0,r) = H_0(r), \quad u(0,r) = u_0(r), \quad p(0) = p_0 \qquad (4)$$

$$\frac{\partial H}{\partial r}(x,0) = 0, \quad \frac{\partial u}{\partial r}(x,0) = 0, \quad v(x,0) = 0$$

$$-\frac{k}{C_p}\frac{\partial H}{\partial r}(x,r_w) = q_w(x) \text{ or } H(x,r_w) = H_w(x),$$

$$u(x,r_w) = 0, \quad v(x,r_w) = 0. \qquad (5)$$

It is shown in paper [1] that the written conditions are enough for solving equations in the region of Ω ($x \geqslant 0$, $0 \leqslant r \leqslant r_w$). However in papers [2, 3] the problem is over-determined by giving the profile $v(0,r)$ in the starting cross-section. Furthermore, in paper [2] the starting cross-section profile $\rho(0,r)$ is given which cannot be taken arbitrary, since the pressure corresponding to density and enthalpy must be constant in the cross-section.

It ought to be explained why exactly the giving of $v(0,r)$ is superfluous. As the solution of our problem (1)–(5) at $x = 0$ cannot be conjugated with an arbitrary flow from the left-hand side, the surface $x = 0$ is to be considered the discontinuity surface. Presenting the equations (1)–(3) in the conservation form

$$\frac{\partial \rho u r}{\partial x} + \frac{\partial \rho v r}{\partial r} = 0 \qquad (1')$$

$$\frac{\partial (p + \rho u^2) r}{\partial x} + \frac{\partial \rho u v r}{\partial r} = \frac{\partial}{\partial r}\left(r\mu \frac{\partial \mu}{\partial r}\right) \qquad (2')$$

$$\frac{\partial \rho u H r}{\partial x} + \frac{\partial \rho v H r}{\partial r} = \frac{\partial}{\partial r}\left(r \frac{k}{C_p}\frac{\partial H}{\partial r}\right) \qquad (3')$$

we can derive from them the conservation conditions which must be true on the discontinuity surface $x = 0$ [4]

$$[\rho u] = 0, \quad [p + \rho u^2] = 0, \quad [\rho u H] = 0. \qquad (6)$$

Consequently, while crossing the surface $x = 0$ the parameter values u, H and p are conserved and only the radial velocity v may have discontinuity.

Since in the problem under consideration $(\partial p/\partial r) = 0$ and $(\mathrm{d}p/\mathrm{d}x) \sim M^2$, then ρ, μ, k, C_p can be assumed to be prescribed temperature functions only. In this case the system of equations (1)–(3) with boundary conditions (5) permits stretching at $q_w = \text{const}$ (or $H_w = \text{const}$):

$$x' = \alpha x, \quad r' = r, \quad u' = \alpha u, \quad v' = v, \quad H' = H,$$

$$\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)' = \alpha \left(\frac{\mathrm{d}p}{\mathrm{d}x}\right) \qquad (7)$$

which allows nondimensional parameters to be introduced, so that the Reynolds number will not enter into the equations and boundary conditions [1]. That means that the obtained solution can be re-evaluated for the flow with similar initial axial velocity profile by means of simple stretching of the x-coordinate.

We point out that equation (1d) included in the initial system of equations in paper [2] is not independent. It may be derived by radial integration of the continuity equation using boundary conditions for v.

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